

**AMENDMENT TO THE CLAIMS**

The following listing of claims replaces all prior versions and listings in the application:

Listing of Claims:

1 - 17. (Canceled)

18. (Currently Amended) A system, comprising:

a memory storing a problem;

a processor coupled to the memory, the processor comprising:

means for constructing a data structure representative of the problem, the data structure comprising ~~nodes representing variables of the problem and constraints of the variables~~ constrained discrete variables of the problem and constraints of the variables, each variable having a referenced set of possible states;

means for delivering messages between a variable of the variables and at least one constraint of the variable, the messages comprising a message containing a set of probabilities for various patterns of warning for the variable, a warning giving information on whether the various assignments from the set of possible states of the variable are compatible with the constraints involving the variable;

means for calculating a probability of the variable satisfying the at least one constraint of the variable based on the messages delivered in order to compute a degree of polarization of the variable to know to what degree the most favorable assignment of the variable is better than all other possible assignments of the variable;

means for assigning at least one of the variable a state from its set of possible states according to the computed degrees of polarization, for simplifying the problem;

means for recursively using means for constructing the data structure, delivering messages, calculating probabilities, and assigning until at least all the variables have been assigned for providing a solution to the problem; and

~~means for providing a solution to the problem based on the probability; and~~

an output device coupled to the processor, the output device outputting the solution of the problem.

19. (Currently Amended) The system of claim 18, the data structure comprising a graph with nodes.

20. (Previously Presented) The system of claim 18, the data structure comprising a list.

21. (Canceled)

22. (Canceled)

23. (Canceled)

24. (Previously Presented) The system of claim 18, the messages comprising a list of binary number equaling the number of possible states of the variable.

25. (Previously Presented) The system of claim 24, the list comprising  $2^q$  entries,  $q$  equaling the number of possible states for the variable.

26. (Canceled)

27. (Currently Amended) The system of claim ~~[[26]]~~ 18, the means for delivering ~~comprise~~ comprising sending warning messages from the at least one constraint to the variable.

28. (Canceled)

29. (Currently Amended) The system of claim ~~[[28]]~~ 18, the means for delivering ~~comprise~~ comprising sending void messages from the at least one constraint to a corresponding variable and indicating that a state of the variable is compatible with the at least one constraint.

30. (Currently Amended) The system of claim ~~[[18]]~~ 31, the messages comprising a list of integer values related to a variable, the list of integer values equaling the number of possible

states of the variable and each integer value totaling the number of warning messages received by the variable from other corresponding constraints.

31. (Currently Amended) The system of claim [[30]] 18, the means for delivering ~~comprise~~ comprising sending an integer value from the variable to the at least one corresponding constraint.

32. (Currently Amended) The system of claim [[18]] 31, the messages comprising a list of integer values related to the variable, the list of integer values equaling the number of possible states of the variable, and each integer value totaling the number of void messages received by the variable from other corresponding constraint, a void message indicating that a state of a variable is compatible with the at least one constraint.

33. (Canceled)

34. (Previously Presented) The system of claim 18, further comprising means for arranging the variables of the problem and the constraints into a truth table to determine a possible state for each variable.

35. (Previously Presented) The system of claim 18, further comprising means for constructing a look-up table comprising the constraints and the variables to determine if the constraints are satisfied.

36. (Previously Presented) The system of claim 35, further comprising means for updating the look-up table to determine a probability that the constraints will not be satisfied for each state of the variables.

37. (Previously Presented) The system of claim 18, further comprising means for assigning a state to the variable when all constraints of the variable is reduced or satisfied.

38. (Previously Presented) The system of claim 18, further comprising means for implementing a penalty function if the variables do not satisfy any constraints, the penalty function determining a probability of satisfying all constraints of each variable in a given state.

39. (Previously Presented) The system of claim 18, further comprising means for randomly choosing a constraint and a corresponding variable to begin solving the problem.

40. (Previously Presented) The system of claim 18, further comprising means for choosing one variable and all corresponding constraints to begin solving the problem.

41. (Currently Amended) A method for solving a problem, comprising:

constructing, in constructing means, a data structure representative of the problem comprising a set of discrete variables, each variable of the set having a referenced set of possible states, each variable of the set having at least one corresponding constraint;

delivering, in delivering means, messages between a variable of the set of variables and the at least one corresponding constraint, messages comprising a message containing a set of probabilities for various patterns of warning for a variable, a warning giving information on whether various assignments from the set of possible states of the variable are compatible with the constraints involving the variable; and

calculating, in calculating means, a set of numbers dependent on the messages, each number in the set of numbers representing a probability of satisfying all constraints of the variable in a given state in order to compute a degree of polarization of the variable to know to what degree the most favorable assignment of the variable is better than all other possible assignment of the variable;

assigning, in assigning means, at least one of the variable a state from its set of possible states according to the computed degrees of polarization, for simplifying the problem; and

recursively using means for constructing the data structure, delivering messages, calculating probabilities and assigning until at least all the variables have been assigned as to provide a solution to the problem.

42. (Currently Amended) The method of claim 41, the data structure comprising a graph with nodes.

43. (Previously Presented) The method of claim 41, the data structure comprising a list.

44. (Canceled)

45. (Canceled)

46. (Previously Presented) The method of claim 41, the messages comprising a list of binary number equaling the number of possible states of the variable.

47. (Previously Presented) The method of claim 46, the list comprising  $2^q$  entries, q equaling the number of states for the variable.

48. (Canceled)

49. (Currently Amended) The method of claim [[48]] 41, the delivering means comprising sending warning messages from the at least one constraint to the variable.

50. (Canceled)

51. (Currently Amended) The method of claim [[50]] 41, the delivering means comprising sending void messages from the at least one constraint to the variable and indicating that a state of the variable is compatible with the at least one constraint.

52. (Currently Amended) The method of claim [[41]] 53, the messages comprising a list of integer values related to the variable, the list of integer values equaling the number of possible states of the variable and each integer value totaling the number of warning messages received by the variable from other corresponding constraints.

53. (Currently Amended) The method of claim [[52]] 41, the delivering means comprising sending an integer value from the variable to the at least one corresponding constraint.

54. (Currently Amended) The method of claim [[41]] 53, the messages comprising a list of integer values related to the variable, the list of integer values equaling the number of possible states of the variable, and each integer value totaling the number of void messages received by the variable from other corresponding constraint, a void message indicating that a state of a variable is compatible with the at least one constraint.

55. (Canceled)

56. (Previously Presented) The method of claim 41, the set of numbers equaling the number of possible states of the variable.

57. (Previously Presented) The method of claim 41, further comprising arranging in arranging means each variable of the set of variables and the at least one constraint into a truth table to determine a possible state for each variable.

58. (Previously Presented) The method of claim 41, further comprising constructing in constructing means a look-up table comprising the at least one constraint and all variables associated with the at least one constraint to determine if the at least constraint is satisfied.

59. (Previously Presented) The method of claim 58, further comprising updating in updating means the look-up table to determine a probability that the at least one constraint will not be satisfied for each state of all variables associated with the at least one constraint.

60. (Previously Presented) The method of claim 41, further comprising assigning in assigning means a state to the variable when all constraints of the variable is reduced or satisfied.

61. (Previously Presented) The method of claim 41, further comprising implementing in implementing means a penalty function if each variable in the set of variables does not satisfy any constraints, the penalty function determining a probability of satisfying all constraints of each variable in a given state.

62. (Previously Presented) The method of claim 41, prior to the step of delivering, randomly choosing in a choosing means a constraint and a corresponding variable to begin solving the problem.

63. (Previously Presented) The method of claim 41, prior to the step of delivering, choosing in a choosing means one variable and all corresponding constraints to begin solving the problem.

64. (Previously Presented) A computer program, comprising computer or machine-readable program elements translatable for implementing the method of claim 41.

65. (Currently Amended) ~~A method for solving a problem, comprising:~~ The method of claim 41, further comprising:

implementing in implementing means a cavity-bias survey, the cavity-bias survey  
sending in sending means messages from a constraint to a corresponding  
variable to determine if the corresponding variable satisfies the constraint;

implementing in implementing means a cavity-field survey, the cavity-field survey sending in sending means messages from the corresponding variable to the constraint to determine if the variable satisfy other constraints related to the variable;

solving in solving means a probability if a given state of the variable satisfy the constraint based on the messages; and

assigning in assigning means a state to the variable from the probability to solve the problem.

66. (Currently Amended) A program storage device readable by a machine, tangibly embodying a program of instructions executable by the machine to perform the method for solving a problem, the method steps comprising:

constructing, in constructing means, a data structure representative of the problem comprising a set of discrete variables, each variable of the set having a referenced set of possible states, each variable of the set having at least one corresponding constraint;

delivering, in delivering means, messages between a variable of the set of variables and [[its]] the at least one corresponding constraint, messages comprising a message containing a set of probabilities for various patterns of warning for the variable, a warning giving information on whether the various assignments from the set of possible states of the variable are compatible with the constraints involving the variable; and

calculating, in calculating means, a set of numbers dependent on the messages, each number in the set of numbers representing a probability of satisfying all constraints of the variable in a given state as to compute a degree of polarization of the variable to know to what degree the most favorable assignment of the variable is better than all other possible assignments of the variable;

assigning, in assigning means, at least one of the variable a state from its set of possible states according to the computed degrees of polarization for simplifying the problem; and

recursively using means for constructing the data structure, delivering messages, calculating probabilities and assigning until at least all the variables have been assigned as to provide a solution to the problem.

67. (New) The system of claim 18, the processor further comprising:

- (a) means for a determining a survey propagation in which the data structure of the problem is constructed and messages are delivered;
- (b) means for determining a survey induced decimation in which, using the previous messages, a degree of polarization for a variable being computed by first establishing a list of numbers, each one giving how favorable it is to assign the variable to one state of its set of possible states, each of these numbers being computed from the probability of warnings determined in the previous messages, the list being used to compute the degree of polarization by determining to what degree the most favorable assignment in the list is better than all other possible assignments, and at least one of the variables is assigned one state of its set of possible states according to its degree of polarization as to simplify the problem; and
- (c) means for restarting from step (a) with a new simplified problem until all variables are either assigned or are unpolarized.

68. (New) The system of claim 67, (a) the means for survey propagation comprising:

determining from a modelization of the problem, an interaction graph with edges linking nodes, the nodes having the variables,  $S_1, S_2, S_3, \dots, S_n$ , and the constraints,  $a, b, c, d, \dots$ , a variable node being connected by an edge to its related constraint(s), an interaction graph being bipartite, a variable being connected only to constraints, a constraint being connected only to variables, and for each constraint which is linked to a variable, updating the graph by:



determining over the graph, first a list of elementary messages called cavity-bias sent from each constraint to its related variable, the cavity-bias being a message having a number of binary items equal to the number of possible states of the variable to which it is sent, each binary item being either void or non-void, the void corresponding to an absence of constraint on the corresponding state of the variable and the non-void corresponding to the reverse, the cavity biases being initialized to random values;

determining over the graph, a list of second elementary messages called cavity-fields sent from each variable to its related constraints, the cavity-field being a message having a number of integer items equal to the number of possible states of the variable which sends it, each integer item value being the number of non-void received from all cavity-biases to the variable for the referenced possible state of the variable;

determining over the graph, a list of local-fields which are sets of integer values in relation to variables, each local-field being a set having a number of integer values equal to the number of possible states of the variable and each integer value being the number of non-void received by the variable in cavity-biases for each possible state of the variable;

the cavity-bias sent from a constraint  $a$  to a variable  $S$  being computed on the constraint  $a$  from at least one cavity-field received by the constraint  $a$  from all the other variables to which the constraint  $a$  is linked, thus excluding  $S$ , and, for each of the cavity-fields, the least penalized subspace of possible states of the variable is determined as being a set of the references of possible states for which the number of non-void is minimal, then a truth table restricted to the sets for all the cavity-fields and for all the references of possible states for the variable  $S$  is created in relation to the constraint  $a$ ,

from this restricted truth table a void is assigned in the cavity-bias for the referenced possible state of the variable  $S$  if the constraint is satisfied and a non-void if the constraint is not satisfied;

determining over the graph, probability laws of each cavity-bias sent from the constraint  $a$  to a linked variable  $S$  with  $q$  possible states and called cavity-bias-surveys, a cavity-bias-survey being a set of  $2^q$  probabilities for each possible configuration of its cavity-bias;

determining over the graph, probability laws of each cavity-field sent from the variable  $S$  to a linked constraint  $a$  and called cavity-field-surveys, a cavity-field-survey being a set of probabilities for each admissible configuration of its cavity-field, an admissible configuration of cavity-field being one with at least one void;

the cavity-bias-survey sent from the constraint  $a$  to the variable  $S$  being computed on the constraint  $a$  from at least one cavity-field-survey received by the constraint  $a$  from all the other variables to which the constraint  $a$  is linked, thus excluding the variable  $S$ , by using a look-up table characterizing constraint  $a$ , the look-up table being a list giving, for each possible assignment of all variables participating to the constraint, whether the constraint is satisfied by the assignment or not, and computing the probability that the constraint is unsatisfied, for each state of the variable;

previous survey propagation steps updates being run successively on the constraints and variables along the graph,

the updates being stopped after a predetermined number of updates if it is not possible to find a set of cavity-bias-surveys which does not change, when one round of updates on all constraints and on all variables participating to the constraints is performed within a given pre-assigned resolution and, then, being restarted from the beginning with cavity biases initialized to new random values;

(b) the means for survey induced decimation comprising:

determining over the graph, a local-field-survey for each variable which is a probability law of all possible local-field by computing for each variable S from all the cavity-bias-surveys received by the variable and for each possible state of the variable the joint probability of each admissible local-field, an admissible local-field being one with at least one zero value, and with the previously determined local-field-surveys;

determining the degree of polarization of each variable by computing, for each assignment of the variable, the probability of having zero value as given by the local-field-survey, and computing for each assignment of the variable, the maximum of this probability diminished by the sum of the probabilities for all other assignments;

where the variable with the largest degree of polarization is assigned to its preferred state, the one with the largest probability of having zero value as given by the local-field-survey;

where the constraints containing this assigned variable are reduced, those which are satisfied are eliminated, in order to make a simpler new problem; and

(c) the means for restarting comprising:

restarting from survey propagation steps (a) with the new problem until all variables are assigned or are unpolarized such that for all the possible assignments of the variable, the probabilities of having zero value as given by the local-field-survey, diminished by the sum of the probabilities of having zero value as given by the local-field-survey for all other assignments, are equal within a predetermined resolution.

69. (New) The method of claim 41, further comprising providing:

- (a) a survey propagation in which the data structure of the problem is constructed and messages are delivered,
- (b) a survey induced decimation in which, using the previous messages, a degree of polarization for a variable being computed by first establishing a list of numbers, each one giving how favorable it is to assign the variable to one state of its set of possible states, each of these numbers being computed from the probability of

warnings determined in the previous messages, the list being used to compute the degree of polarization by determining to what degree the most favorable assignment in the list is better than all other possible assignments, and at least one of the variables is assigned one state of its set of possible states according to its degree of polarization as to simplify the problem,

- (c) an iteration for restarting from step (a) with the new simplified problem till all variables are either assigned or are unpolarized.

70. (New) The method of claim 69, (a) the survey propagation comprising:

determining from a modelization of the problem, an interaction graph with edges linking nodes, the nodes being the variables,  $S_1, S_2, S_3, \dots$ , and the constraints,  $a, b, c, d, \dots$ , a variable node being connected by an edge to its related constraint(s), an interaction graph being bipartite, a variable being connected only to constraints, a constraint being connected only to variables, and for each constraint which is linked to a variable, updating the graph by:

determining over the graph, first a list of elementary messages called cavity-bias sent from each constraint to its related variable, the cavity bias being a message having a number of binary items equal to the number of possible states of the variable to which it is sent, each binary item being either void or non-void, the void corresponding to an absence of constraint on the corresponding state of the variable and the non-void corresponding to the reverse, the cavity biases being initialized to random values;

determining over the graph, a list of second elementary messages called cavity-fields sent from each variable to its related constraints, the cavity-field being a message having a number of integer items equal to the number of possible states of the variable which sends it, each integer item value being the number of non-void received from all cavity-biases to the variable for the referenced possible state of the variable;

determining over the graph, a list of local-fields which are sets of integer values in relation to variables, each local-field being a set having a number of integer values equal to the number of possible states of the variable and each integer value being the number of non-void received by the variable in cavity-biases for each possible state of the variable;

the cavity-bias sent from the constraint  $a$  to the variable  $S$  being computed on the constraint  $a$  from the cavity-field(s) received by the constraint  $a$  from all the other variables to which the constraint  $a$  is linked, thus excluding the variable  $S$ , and, for each of the cavity-fields, the least penalized subspace of possible states of the variable is determined as being a set of the references of possible states for which the number of non-void is minimal, then a truth table restricted to the sets for all the cavity-fields and for all the references of possible states for the variable  $S$  is created in relation to the constraint  $a$ , from this restricted truth table a void is assigned in the cavity-bias for the referenced possible state of the variable  $S$  if the constraint is satisfied and a non-void if the constraint is not satisfied;

determining over the graph, probability laws of each cavity-bias sent from the constraint  $a$  to a linked variable  $S$  with  $q$  possible states and called cavity-bias-surveys, a cavity-bias-survey being a set of  $2^q$  probabilities for each possible configuration of its cavity-bias;

determining over the graph, probability laws of each cavity-field sent from the variable  $S$  to a linked constraint  $a$  and called cavity-field-surveys, a cavity-field-survey being a set of probabilities for each admissible configuration of its cavity-field, an admissible configuration of cavity-field being one with at least one void;

the cavity-bias-survey sent from the constraint  $a$  to the variable  $S$  being computed on the constraint  $a$  from at least one cavity-field-survey received by the constraint  $a$  from all the other variables to which the constraint  $a$  is linked, thus excluding the variable  $S$ , by using a look-up table characterizing

constraint  $a$ , the look-up table being a list giving, for each possible assignment of all variables participating to the constraint, whether the constraint is satisfied by the assignment or not, and computing the probability that the constraint is unsatisfied, for each state of the variable,

previous survey propagation steps updates being run successively on the constraints and variables along the graph,

the updates being stopped after a predetermined number of updates if it is not possible to find a set of cavity-bias-surveys which does not change, when one round of updates on all constraints and on all variables participating to the constraints is performed within a given preassigned resolution and, then, being restarted from the beginning with cavity biases initialized to new random values,

(b) the survey induced decimation comprising:

determining over the graph, a local-field-survey for each variable which is a probability law of all possible local-field by computing for each variable  $S$  from all the cavity-bias-surveys received by the variable and for each possible state of the variable the joint probability of each admissible local-field, an admissible local-field being one with at least one zero value, and with the previously determined local-field-surveys:

determining the degree of polarization of each variable by computing, for each assignment of the variable, the probability of having zero value as given by the local-field-survey, and computing for each assignment of the variable, the maximum of this probability diminished by the sum of the probabilities for all other assignments;

where the variable with the largest degree of polarization is assigned to its preferred state, the one with the largest probability of having zero value as given by the local-field-survey;

where the constraints containing this assigned variable are reduced, those which are satisfied are eliminated, in order to make a simpler new problem; and

(c) the iteration comprising:

restarting from survey propagation steps (a) with the new problem until all variables are assigned or are unpolarized such that, for all the possible assignments of the variable, the probabilities of having zero value as given by the local-field-survey, diminished by the sum of the probabilities of having zero value as given by the local-field-survey for all other assignments, are equal within a predetermined resolution.